

GEOMETRICALLY FINITE AND INFINITE KLEINIAN GROUPS

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ABSTRACT. This is a summary of the material for 3 lectures on geometrically finite and infinite Kleinian groups delivered by the author at a conference held at Tata Institute of Fundamental Research in April 2014.

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1. LECTURE 1: GEOMETRICALLY FINITE GROUPS

1.1. **Fuchsian Groups.** A Kleinian group G is a discrete subgroups of $PSL_2(\mathbb{C}) = Mob(\widehat{\mathbb{C}}) = Isom(\mathbb{H}^3)$. This gives us three closely intertwined perspectives on the field:

- (1) Studying discrete subgroups G of the group of Mobius transformations $Mob(\widehat{\mathbb{C}})$ emphasizes the *Complex Analytical/Dynamic* aspect.
- (2) Studying discrete subgroups G of $PSL_2(\mathbb{C})$ emphasizes the *Lie group/matrix group* theoretic aspect.
- (3) Studying discrete subgroups G of $Isom(\mathbb{H}^3)$ emphasizes the *Hyperbolic Geometry* aspect.

We shall largely emphasize the third perspective. Since G is discrete, we can pass to the quotient $M^3 = \mathbb{H}^3/G$. Thus we are studying hyperbolic structures on 3-manifolds.

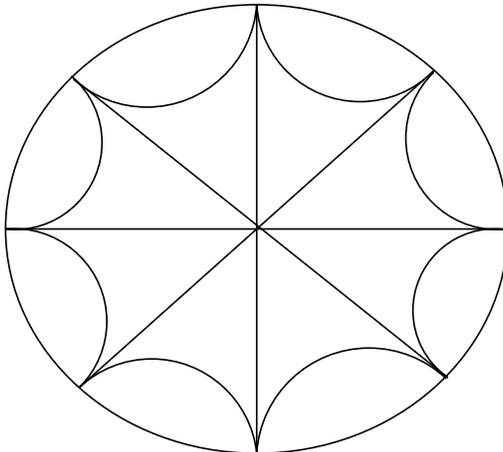
Date: July 16, 2015.

2010 *Mathematics Subject Classification.* 57M50, 20F67 (Primary); 20F65, 22E40 (Secondary).

In order to obtain some examples, we first move one dimension down and look at discrete subgroups G of the group of Möbius transformations $Mob(\Delta) = Mob(\mathbb{H})$ of the unit disk (which is conformally equivalent to the upper half plane). These are called Fuchsian Groups, and were discovered by Poincaré. The natural metric of constant negative curvature on the upper half plane is given by $ds^2 = \frac{dx^2+dy^2}{y^2}$. This is called the *hyperbolic metric*. The resulting space is denoted as \mathbb{H}^2 .

The associated conformal structure is exactly the complex structure on $\mathbb{H} = \{z \in \mathbb{C} : \text{Im}(z) > 0\}$. It turns out that orientation preserving isometries of \mathbb{H}^2 are exactly the conformal automorphisms of \mathbb{H}^2 . The boundary circle S^1 compactifies Δ . This has a geometric interpretation. It codes the ‘ideal’ boundary of \mathbb{H}^2 , consisting of asymptote classes of geodesics. The topology on S^1 is induced by a metric which is defined as the angle subtended at $0 \in \Delta$. The geodesics turn out to be semicircles meeting the boundary S^1 at right angles.

We now proceed to construct an example of a discrete subgroup of $Isom(\mathbb{H}^2)$. The genus two orientable surface can be described as a quotient space of an octagon with edges labelled $a_1, b_1, a_1^{-1}, b_1^{-1}, a_2, b_2, a_2^{-1}, b_2^{-1}$, where the boundary has the identification induced by this labelling. In order to construct a metric of constant negative curvature on it, we have to ensure that each point has a small neighborhood isometric to a small ball in \mathbb{H}^2 . To ensure this it is enough to do the above identification on a regular hyperbolic octagon (all sides and all angles equal) such that the sum of the interior angles is 2π . To ensure this, we have to make each interior angle equal $\frac{2\pi}{8}$. The infinitesimal regular octagon at the tangent space to the origin has interior angles equal to $\frac{3\pi}{4}$. Also the ideal regular octagon in \mathbb{H}^2 has all interior angles zero. See figure below.

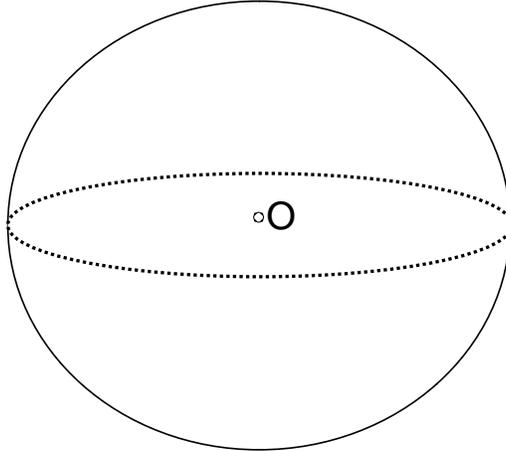


Hence by the Intermediate value Theorem, as we increase the size of the octagon from an infinitesimal one to an ideal one, we shall hit interior angles all equal to $\frac{\pi}{4}$ at some stage. The group G that results from side-pairing transformations corresponds to a Fuchsian group, or equivalently, a discrete faithful representation of the fundamental group of a genus 2 surface into $Isom(\mathbb{H}^2)$. We let ρ denote the associated representation.

1.2. Kleinian Groups. We now move back to \mathbb{H}^3 . The hyperbolic metric is given by $ds^2 = \frac{dx^2+dy^2+dz^2}{z^2}$ on upper half space. Note that the metric blows up as one approaches $z = 0$. Equivalently we could consider the ball model, where the

boundary $S^2 = \widehat{\mathbb{C}}$ consists of ideal end-points of geodesic rays as before. The metric on $\widehat{\mathbb{C}}$ is given by the angle subtended at $0 \in \mathbb{H}^3$.

Since $Isom(\mathbb{H}^2) \subset Isom(\mathbb{H}^3)$, we can look upon the discrete group G we constructed above also as a discrete subgroup of $Isom(\mathbb{H}^3)$.



In the above picture two things need to be observed.

- 1) the orbit $G.o$ accumulates on the equatorial circle. This is called the *limit set* Λ_G .
- 2) The complement of Λ_G consists of two round open discs. On each of these discs, G acts freely (i.e. without fixed points) properly discontinuously, by conformal automorphisms. Hence quotient is two copies of the ‘same’ Riemann surface (i.e. a one dimensional complex analytic manifold). The complement $\widehat{\mathbb{C}} \setminus \Lambda_G = \Omega_G$ is called the *domain of discontinuity* of G .

We proceed with slightly more formal definitions identifying $\widehat{\mathbb{C}}$ with the sphere S^2 .

Definition 1.1. *If $x \in \mathbf{H}^3$ is any point, and G is a discrete group of isometries, the limit set $\Lambda_G \subset S^2$ is defined to be the set of accumulation points of the orbit $G.x$ of x .*

The domain of discontinuity for a discrete group G is defined to be $\Omega_G = S^2 \setminus \Lambda_G$.

Proposition 1.2. [Thu80][Proposition 8.1.2] *If G is not elementary, then every non-empty closed subset of S^2 invariant by G contains the limit set Λ_G .*

Suppose that G is abstractly isomorphic to the fundamental group of a finite area hyperbolic surface S^h , and $\rho : \pi_1(S^h) \rightarrow PSL_2(\mathbb{C})$ be a representation with image G . Suppose further that ρ is *strictly type-preserving*, i.e. $g \in \pi_1(S^h)$ represents an element in a peripheral (cusp) subgroup if and only $\rho(g)$ is parabolic. In this situation we shall refer to G as a *surface Kleinian group*. A recurring theme in the context of finitely generated, infinite covolume Kleinian groups is that the general theory can be reduced to the study of surface Kleinian groups. Equivalently, we study the representation space $Rep(\pi_1(S^h), PSL_2(\mathbb{C}))$.

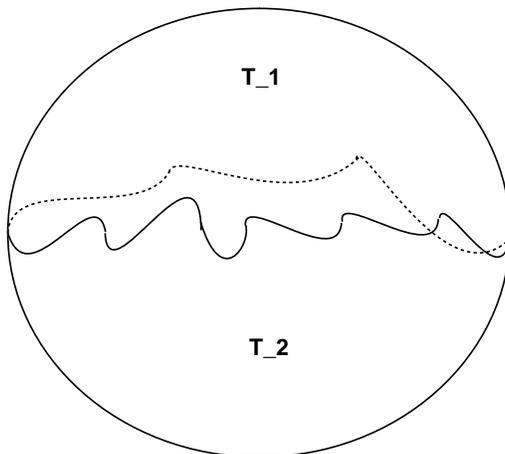
Regarding G as a subgroup of $Mob(\widehat{\mathbb{C}})$, the *dynamics* of the action of G on $\widehat{\mathbb{C}}$ emerges. The *limit set* Λ_G of G is defined to be the set of accumulation points of the orbit $G.o$ in $\widehat{\mathbb{C}}$ for some (any) $o \in \mathbb{H}^3$. The limit set is the locus of chaotic

dynamics of the action of G on \mathbb{C} . The complement $\widehat{\mathbb{C}} \setminus \Lambda_G = \Omega_G$ is called the *domain of discontinuity* of G .

On the other hand regarding G as a subgroup of $Isom(\mathbb{H}^3)$, we obtain a quotient hyperbolic 3-manifold $M = \mathbb{H}^3/G$ with fundamental group G .

A major problem in the theory of Kleinian groups is to **understand the relationship between the dynamic and the hyperbolic geometric descriptions** of G .

The **Ahlfors-Bers simultaneous Uniformization Theorem** states that given any two conformal structures τ_1, τ_2 on a surface, there is a discrete subgroup G of $Mob(\widehat{\mathbb{C}})$ whose limit set is *topologically* a circle, and whose domain of discontinuity quotients to the two Riemann surfaces τ_1, τ_2 . See figure below.



The limit set is a quasiconformal map of the round circle. These (*quasi Fuchsian*) groups can be thought of as *deformations* of Fuchsian groups (Lie group theoretically) or quasiconformal deformations (analytically). Ahlfors and Bers proved that these are precisely all quasiconvex surface Kleinian groups.

The convex hull CH_G of Λ_G is the smallest closed convex subset of \mathbb{H}^3 invariant under G . It can be constructed by joining all pairs of points on limit set by bi-infinite geodesics and iterating this construction. The quotient of CH_G by G , which is homeomorphic to $S^h \times [0, 1]$, is called the *Convex core* $CC(M)$ of $M = \mathbb{H}^3/G$.

The ‘thickness’ of $CC(M)$ for a quasi Fuchsian surface Kleinian group, measured by the distance between $S^h \times \{0\}$ and $S^h \times \{1\}$ is a geometric measure of the complexity of the quasi Fuchsian group G .

2. LECTURE 2: LAMINATIONS

The main technical tools required to deal with the notions of convex hulls and bending introduced In Lecture 1 are laminations and pleated surfaces. I followed Thurston’s (unpublished) notes [Thu80] on the subject.

Definition 2.1. *A geodesic lamination on a hyperbolic surface is a foliation of a closed subset with geodesics.*

Geodesic laminations arise naturally in a number of contexts in the study of hyperbolic 2- and 3- manifolds.

- (1) as stable and unstable laminations corresponding to a pseudo-Anosov diffeomorphism of a hyperbolic surface.
- (2) as the pleating locus of a component of the convex core boundary $\partial CC(M)$ of a hyperbolic 3-manifold M .
- (3) as the ending lamination corresponding to a geometrically infinite end of a hyperbolic 3-manifold.

We shall in this section discuss briefly how each of these examples arise.

2.1. Stable and Unstable laminations. We consider the torus T^2 equipped with a diffeomorphism ϕ , whose action on homology is given by a 2×2 matrix with irrational eigenvalues, e.g. $\frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$. Then the eigendirections give rise to two sets of foliations by dense copies of \mathbb{R} : the stable and unstable foliation. Such a diffeomorphism is called Anosov. Anosov diffeomorphisms of the torus may be characterized in terms of their action on $\pi_1(T)$ as not having periodic conjugacy classes.

Now consider the stable (or unstable) foliation (minus a point $*$) on $S = (T^2 \setminus \{*\})$. Equip S with a complete hyperbolic structure of finite volume and straighten every leaf of the foliation to a complete geodesic. The resulting union of leaves is called the stable (or unstable) lamination of the diffeomorphism ϕ on the hyperbolic surface S .

One of the fundamental pieces of Thurston's work [FLP79] shows that the existence of such a stable and unstable lamination generalizes to all hyperbolic surfaces. A diffeomorphism ϕ of a hyperbolic surface S preserving punctures (or boundary components according to taste) is called pseudo Anosov if the action of ϕ_* on $\pi_1(S)$ has no periodic conjugacy classes. Thurston proved the existence of a unique stable and unstable lamination without any closed leaves for any pseudo Anosov diffeomorphism ϕ acting on a hyperbolic surface S .

2.2. Pleating locus. We quote a picturesque passage from [Thu80]:

Consider a closed curve σ in Euclidean space, and its convex hull $H(\sigma)$. The boundary of a convex body always has non-negative Gaussian curvature. On the other hand, each point p in $\partial H(\sigma) \setminus \sigma$ lies in the interior of some line segment or triangle with vertices on σ . Thus, there is some line segment on $\partial H(\sigma)$ through p , so that $\partial H(\sigma)$ has non-positive curvature at p . It follows that $\partial H(\sigma) \setminus \sigma$ has zero curvature, i.e., it is developable. If you are not familiar with this idea, you can see it by bending a curve out of a piece of stiff wire (like a coathanger). Now roll the wire around on a big piece of paper, tracing out a curve where the wire touches. Sometimes, the wire may touch at three or more points; this gives alternate ways to roll, and you should carefully follow all of them. Cut out the region in the plane bounded by this curve (piecing if necessary). By taping the paper together, you can envelope the wire in a nice paper model of its convex hull. The physical process of unrolling a developable surface onto the plane is the origin of the notion of the developing map.

The same physical notion applies in hyperbolic three-space. If K is any closed set on S^2 (the sphere at infinity), then $H(K)$ is convex, yet each point on $\partial H(K)$ lies on a line segment in $\partial H(K)$.

Thus, $\partial H(K)$ can be developed to a hyperbolic plane. (In terms of Riemannian geometry, $\partial H(K)$ has extrinsic curvature 0, so its intrinsic curvature is the ambient sectional curvature, -1. Note however that $\partial H(K)$ is not usually differentiable). Thus $\partial H(K)$ has the natural structure of a complete hyperbolic surface.

This forces $\partial H(K)$ equipped with its intrinsic metric to be a hyperbolic surface. However, there are complete geodesics along which it is bent (but not crumpled). Thus each boundary component S , and hence its universal cover \tilde{S} , carries a metric that is intrinsically hyperbolic. However, in \mathbf{H}^3 , the universal cover \tilde{S} is bent along a geodesic lamination. S is an example of a **pleated surface**:

Definition 2.2. [Thu80][Definition 8.8.1] *A **pleated surface** in a hyperbolic three-manifold N is a complete hyperbolic surface S of finite area, together with an isometric map $f : S \rightarrow N$ such that every $x \in S$ is in the interior of some straight line segment which is mapped by f to a straight line segment. Also, f must take every cusp of S to a cusp of N*

The pleating locus of the pleated surface $f : S \rightarrow M$ is the set $\gamma \subset S$ consisting of those points in the pleated surface which are in the interior of unique line segments mapped to line segments.

Proposition 2.3. [Thu80][Proposition 8.8.2] *The pleating locus γ is a geodesic lamination on S . The map f is totally geodesic in the complement of γ .*

2.3. Ending Laminations. The notion of an ending lamination comes up in the context of a geometrically infinite group. We shall deal with these groups in greater detail in Lecture 3. Thurston introduces the notion of a geometrically tame end E of a manifold M . An end E of a hyperbolic manifold M is geometrically tame (and geometrically infinite) if there exists a sequence of pleated surfaces exiting E .

For such an end E , choose a sequence of simple closed curves $\{\sigma_n\}$ exiting E . Let $S = \partial E$ be the bounding surface of E . Then the limit of such a sequence (in a suitable sense; the reader will not be much mistaken if (s)he thinks of the Hausdorff limit on the bounding surface S of E) is a lamination λ . It turns out that λ is independent of the sequence $\{\sigma_n\}$.

3. LECTURE 3: GEOMETRICALLY INFINITE GROUPS

3.1. Degenerate Groups. The most intractable examples of surface Kleinian groups are obtained as limits of quasi Fuchsian groups. In fact, it has been recently established by Minsky et al. [Min10] [BCM12] that the set of all surface Kleinian groups (or equivalently all discrete faithful representations of a surface group in $PSL_2(\mathbb{C})$) are given by quasiFuchsian groups and their limits. This is known as the Bers density conjecture.

To construct limits of quasi Fuchsian groups, one allows the thickness of the convex core $CC(M)$ to tend to infinity. There are two possibilities:

- a) Let only τ_1 degenerate. i.e. $I \rightarrow [0, \infty)$ (**simply degenerate case**)
- b) Let both τ_1, τ_2 degenerate, i.e. $I \rightarrow (-\infty, \infty)$ (**doubly degenerate case**)

Thurston's Double Limit Theorem [Thu86] says that these limits exist. A fundamental question in **relating the geometric and dynamic aspects** of Kleinian groups is the following.

Question 3.1. (*Thurston*) *How does the limit set behave for the limiting manifold?*

In the doubly degenerate case the limit set is all of \widehat{C} .

In the next section we outline our approach and solution to this problem.

3.2. Extensions of Maps to Ideal Boundaries. Starting with [Mit98b], [Mit98a] and [Mit97a], we investigated the following question:

Question 3.2. *Let G be a hyperbolic group in the sense of Gromov acting freely and properly discontinuously by isometries on a hyperbolic metric space X . Does the inclusion of the Cayley graph $i : \Gamma_G \rightarrow X$ extend continuously to the (Gromov) compactifications?*

A positive answer to Question 3.2 gives us a precise handle on Question 3.1. In this generality the question first appears in [Mit97b] (see also the Geometric Group Theory Problem List [Bes04]). As of date no counterexample is known.

However, special cases of Question 3.2 have been raised earlier in the context of Kleinian groups.

•1 In Section 6 of [CT85] (now published as [CT07]), Cannon and Thurston propose the following.

Conjecture 3.3. *Suppose a surface group $\pi_1(S)$ acts freely and properly discontinuously on \mathbb{H}^3 by isometries. Then the inclusion $\tilde{i} : \widetilde{S^h} \rightarrow \mathbb{H}^3$ extends continuously to the boundary*

The authors of [CT85] point out that for a simply degenerate group, this is equivalent to asking if the limit set is locally connected.

•2 In [McM01], McMullen makes the following more general conjecture:

Conjecture 3.4. *For any hyperbolic 3-manifold N with finitely generated fundamental group, there exists a continuous, $\pi_1(N)$ -equivariant map*

$$F : \partial\pi_1(N) \rightarrow \Lambda \subset S_\infty^2$$

where the boundary $\partial\pi_1(N)$ is constructed by scaling the metric on the Cayley graph of $\pi_1(N)$ by the conformal factor of $d(e, x)^{-2}$, then taking the metric completion. (cf. Floyd [Flo80])

In [Mj14a] and [Mj10b] we provide a complete positive answer to both Conjectures 3.3 and 3.4.

As a consequence we also establish in [Mj14a] the following Theorem which proves a long-standing conjecture in the theory of Kleinian groups [Abi76] [CT85].

Theorem 3.5. *Connected limit sets of finitely generated Kleinian groups is locally connected.*

In the next subsection, after describing the history of these problems, we shall give more details about the structure of limit sets and their relation to the geometry of surface Kleinian groups.

3.3. History and Solution of the Problem. In [Abi76], Abikoff (1976) claimed to prove that limit sets of simply degenerate surface Kleinian groups were never locally connected. Thurston and Kerckhoff found a flaw in his proof in about 1980.

The first major result that started this entire program was Cannon and Thurston's result [CT85] for hyperbolic 3-manifolds fibering over the circle with fiber a closed surface group.

Let M be a closed hyperbolic 3-manifold fibering over the circle with fiber F . Let \widetilde{F} and \widetilde{M} denote the universal covers of F and M respectively. Then \widetilde{F} and \widetilde{M} are quasi-isometric to \mathbb{H}^2 and \mathbb{H}^3 respectively. Now let $\mathbb{D}^2 = \mathbb{H}^2 \cup \mathbb{S}_\infty^1$ and $\mathbb{D}^3 = \mathbb{H}^3 \cup \mathbb{S}_\infty^2$ denote the standard compactifications. In [CT85] Cannon and Thurston show that the usual inclusion of \widetilde{F} into \widetilde{M} extends to a continuous map from \mathbb{D}^2 to \mathbb{D}^3 . This was extended to Kleinian surface groups of bounded geometry without parabolics by Minsky [Min94].

An alternate approach (purely in terms of coarse geometry ignoring all local information) was given by the author in [Mit98b] generalizing the results of both Cannon-Thurston and Minsky. We proved the Cannon-Thurston result for hyperbolic 3-manifolds of bounded geometry without parabolics and with freely indecomposable fundamental group. A different approach based on Minsky's work was given by Klarreich [Kla99].

Bowditch [Bow07] [Bow02] proved the Cannon-Thurston result for punctured surface Kleinian groups of bounded geometry. In [Mj09] we gave an alternate proof of Bowditch's results and simultaneously generalized the results of Cannon-Thurston, Minsky, Bowditch, and those of [Mit98b] to all 3 manifolds of bounded geometry whose cores are incompressible away from cusps. The proof has the advantage that it reduces to a proof for manifolds without parabolics when the 3 manifold in question has freely indecomposable fundamental group and no accidental parabolics.

In the expository paper [Mj10a] we give our proof of the results of Cannon and Thurston [CT85], Minsky [Min94], and Bowditch [Bow07] using the ideas of [Mit98b] and [Mj09].

In [Min99] Minsky established a bi-Lipschitz model for all punctured torus Kleinian groups. McMullen [McM01] proved the Cannon-Thurston result for punctured torus groups, using Minsky's model for these groups [Min99].

In [Mj11] we identified a large-scale coarse geometric structure involved in the Minsky model for punctured torus groups (and called it **i-bounded geometry**). *i-bounded geometry* can roughly be regarded as that geometry of ends where the boundary tori of Margulis tubes have uniformly bounded diameter. We gave a proof for models of *i-bounded geometry*. In combination with the methods of [Mj09] this was enough to bring under the same umbrella all known results on Cannon-Thurston maps for 3 manifolds whose cores are incompressible away from cusps.

In [Mj05] we further generalized possible geometries allowing us to push our techniques through to establish the Cannon-Thurston property.

In the mean time, in the proof of the celebrated Ending Lamination Conjecture, Minsky [Min10] and Brock-Canary-Minsky [BCM12] established a bi-Lipschitz model for all surface Kleinian groups.

In [Mj14a], we used the Minsky model of [Min10] to prove that all hyperbolic 3-manifolds homotopy equivalent to a surface satisfy the conditions imposed in the geometries dealt with in [Mj05]. This establishes the Cannon-Thurston property

for all surface Kleinian groups and proves Conjecture 3.3. It follows that surface Kleinian groups have locally connected limit sets. Combining this result with a reduction Theorem of Anderson and Maskit [AM96], we prove that connected limit sets of finitely generated Kleinian groups are locally connected (Theorem 3.5). Finally in [Mj10b] we extend the techniques of [Mj14a] to cover handlebody groups and prove Conjecture 3.4.

We then gave explicit descriptions of the boundary identifications of [Mj14a] in terms of *ending laminations* in [Mj14b]. This finally yields a rather complete and satisfactory solution to Question 3.1.

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