Friday, 16 December 2005 (9.30-10.10)

Speaker: W.M. Schmidt, University of Colorado, Boulder, USA
Title: The Number of Solutions of Exponential Equations

Friday, 16 December 2005 (10.15-10.55)

Speaker: M. Mignotte, Université Louis Pasteur, FRANCE
Title: Linear forms in two or three logarithms

We present the best known estimates on lower bounds of linear forms in two or three logarithms of algebraic numbers. The importance of zero estimates (of Yu. Nesterenko and M. Laurent) is explained. We also discuss briefly on the technical parameters which give some improvements in the case of "small" logarithms (an idea introduced by T.N. Shorey). Several recent examples of applications of these estimates to Diophantine problems will be given.

Friday, 16 December 2005 (11.30-12.10)

Speaker: K. Győry, University of Debrecen, HUNGARY
Title: Binomial Thue equations, S-unit equations and polynomial powers

Let \( p_1, \ldots, p_s \) be primes, and \( S \) the set of integers not divisible by primes different from \( p_1, \ldots, p_s \). As a common generalization of S-unit equations and binomial Thue equations with unknown degree, we consider the equation

\[
ax^n - by^n = c \quad \text{in integers } a, b, c, x, y, n, \]  

where \( a, b, c \in S, n \geq 3 \) and \( \gcd(ax^n, by^n, c) = 1 \).

First, effective finiteness theorems will be presented for (1) in a quantitative form (joint work with I. Pink and A. Pintér (2004)). Then, for \( c = \pm 1 \), \( s = 2 \) and \( p_1, p_2 \leq 13 \), all solutions will be explicitly listed (joint work with M.A. Bennett, M. Mignotte and A. Pintér (200?)). Finally, some applications will be discussed to diophantine equations of the form \( f(x) = wy^n \), where \( f \in \mathbb{Z}[X] \).
Speaker: Noriko HIRATA-Kohno, Nihon University, JAPAN
Title: An absolute upper bound for the number of the integer solutions to $a^x + b^y = c^z$

Let us consider the exponential diophantine equation $a^x + b^y = c^z$ where $a, b, c \geq 2$ are fixed rational integers relatively co-prime and $x, y, z \geq 2$ viewed as unknowns in rational integers. We obtained an absolute upper bound for the number of solutions to this equation. The bound is independent of $a, b, c$. We got the bound by using unit equations and a method coming from transcendence. We give also a bound for the height of the solutions using the theory of linear forms in $m$- logarithms where $m$ is an integer not necessarily a prime.

Speaker: C.L. Stewart, University of Waterloo, CANADA
Title: On heights of multiplicatively dependent algebraic numbers

Our objective in this talk is to prove that if $A$ is a set of non-zero algebraic numbers, any $t$ of which are multiplicatively dependent, then, provided that the cardinality of $A$ is large enough, two of the numbers will have a quotient of small height relative to the heights of other elements of $A$. We shall also discuss some applications of this result.

Speaker: K. Yu, HongKong University of Science and Technology, HONG KONG
Title: Bounds for the Solutions of S-unit Equations

This is a joint work with Prof. K. Györy. I shall report on our new and completely explicit bounds for the solutions of S-unit equations. If time permits, I shall also present our new bounds for decomposable form equations.
Title: Polyadic codes

Polyadic codes constitute a special class of cyclic codes and are generalizations of quadratic residue codes, duadic codes, triadic codes, $m$-adic residue codes and split group codes, which have good error-correcting properties. We give a necessary and sufficient condition for the existence of polyadic codes or $m$-adic codes of prime power length. Polyadic codes are completely characterized in terms of their idempotent generators. We also provide an efficient method to compute their explicit expressions.

Title: Art of Research, Public Lecture
Speaker: M. Ram Murty

Title: To be announced

Speaker: D. Thakur, University of Arizona, USA
Title: Diophantine approximation and transcendence in finite characteristic

Unlike the number field case where Roth’s theorem holds for diophantine approximation of algebraic numbers, its analog fails in function fields, and situation is not even conjecturally understood. We describe the partial progress on this and related issue of continued fractions for algebraics. We will also describe progress on transcendence and algebraic independence issues using higher dimensional generalizations of Drinfeld modules.
Speaker: A. Schinzel, University of Warszawa, POLAND  
Title: On the set of prime divisors of a linear recurrence  

Let for an integer $l \neq 0, \pm1$, $P(l)$ be the greatest prime factor of $l$, $P(0) = 0$, $P(\pm1) = 1$ and for $r = l/m$, $(l, m) = 1$, $P(r) = P(l)$. Let $u_n$ be a linear recurrence of order $k$ defined over $\mathbb{Q}$ with infinitely many terms different from 0. The following theorems hold.

**Theorem 1.** $\limsup P(u_n) < \infty$, if and only if there exists a positive integer $l$ such that for $n < k + l \min\{k, l - 1\}$ the sequence $u_{n+l}/u_n$ is purely periodic with the period of $l$ terms, some of which may be 0/0.

**Theorem 2.** The number $l$ in Theorem 1, if it exists, can be chosen at most $\exp(3.16k\sqrt{\log k})$ and, if the companion polynomial of $u_n$ is irreducible and $k \geq 2$, then at most $e^{3/2k^{3/2}(\log \log k + 4)^{3/2}}$, where $\gamma$ is Euler’s constant.

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Speaker: F. Luca, Instituto de Matematicas UNAM, MEXICO  
Title: Perfect powers in products of terms of Lucas sequences  

Let $(u_n)_{n \geq 0}$ be a Lucas sequence. In my talk, I will discuss various recent results on diophantine equations of the type

$$u_{n_1} \cdot u_{n_2} \cdot \ldots \cdot u_{n_k} = \ell y^m$$

in positive integers $n_1, n_2, \ldots, n_k, |y|, \ell$ and $m \geq 2$, where the largest prime factor of $\ell$ is bounded. For example, I will show that the above equation has no solution is $u_n = (x^n - 1)/(x - 1)$ with a positive integer $x$ when $\ell = 1$, $k > 1$ and $1 < n_1, \ldots, n_k$ are consecutive. I will also list all the solutions of the above equation when $u_n = F_n$ is the $n$th Fibonacci number, $m = p > k$ is prime, and the largest prime factor of $\ell$ does not exceed 541.

These results have been obtained in the past few years in joint works with Y. Bugeaud, M. Mignotte, T.N. Shorey, and S. Siksek.
Speaker: M. Waldschmidt, Universit Pierre et Marie Curie (PARIS VI), FRANCE
Title: The role of complex conjugation in transcendental number theory

In his two well known 1968 papers "Contributions to the theory of transcendental numbers", K. Ramachandra proved several results showing that, in certain explicit sets \( \{x_1, \ldots, x_n\} \) of complex numbers, one element at least is transcendental. In specific cases the number \( n \) of elements in the set was 2 and the two numbers \( x_1, x_2 \) were both real. He then noticed that the conclusion is equivalent to say that the complex number \( x_1 + ix_2 \) is transcendental.

In his 2004 paper published in the Journal de Théorie des Nombres de Bordeaux, G. Diaz investigates how complex conjugation can be used for the transcendence study of the values of the exponential function. For instance, if \( \log \alpha_1 \) and \( \log \alpha_2 \) are two nonzero logarithms of algebraic numbers, one of them being either real or purely imaginary, and not the other, then the product \( (\log \alpha_1)(\log \alpha_2) \) is transcendental.

We survey Diaz results and produce further similar ones.

Speaker: B. Sury, Indian Statistical Institute, Bangalore, INDIA
Title: Quadratic factors of \( f(x) - g(y) \) in positive characteristic

Saturday, 17 December 2005 (4.30-5.10)

Speaker: M. Kulkarni, Indian Statistical Institute, Bangalore, INDIA
Title: Some applications of the Bilu-Tichy theorem to Diophantine equations

Saturday, 17 December 2005 (6-8)

Dance Recital by Students of Nritya Gitanjali
Sunday, 18 December 2005 (9.15-9.55)

Speaker: T.D. Wooley, University of Michigan, USA
Title: The density of rational points on large dimensional hypersurfaces

Methods of Birch (1961) and Schmidt (1985) establish lower bounds for the number of rational points of bounded height on hypersurfaces with dimension sufficiently large in terms of their degree. While Birch’s method requires the codimension of the singular locus of the hypersurface to be large, Schmidt’s approach provides unconditional conclusions when the dimension of that hypersurface is extremely large. We discuss an alternative strategy based on a diagonalization argument combined with the circle method that in many instances improves on Schmidt’s result, and is sufficiently flexible to find application elsewhere.

Sunday, 18 December 2005 (10.00-10.40)

Speaker: S.D. Adhikari, Harish-Chandra Research Institute, Allahabad, INDIA
Title: On some generalizations of Classical zero-sum problems

A well known result of Erdős, Ginzburg and Ziv (EGZ theorem), a prototype of zero-sum theorems in Combinatorial Number Theory, continues to play and important role in the development of this area. Starting with some discussions on the EGZ theorem, we proceed to discuss some of its generalizations, including a very recent one.

Sunday, 18 December 2005 (11.15-11.55)

Speaker: A. Sankaranarayanan, TIFR, Mumbai, INDIA
Title: The sum involving the derivative of $ζ(s)$ over simple zeros

Let $\rho = \beta + i\gamma$ denote the zeros of the Riemann zeta-function $ζ(s)$. In this talk, we discuss how big the sum

$$S := \sum_{|\gamma| \leq T} |ζ^{(1)}(\rho)|^{-1}$$

could be as a function of $T$, assuming only that all the zeros $\rho$ of $ζ(s)$ are simple. It is a joint work with M.Z. Garaev.
Sunday, 18 December 2005 (12.00-12.40)

Speaker: R. Balasubramanian, Institute of Mathematical Sciences, Chennai, INDIA
Title: To be announced

Sunday, 18 December 2005 (12.45-1.25)

Speaker: K. Ramachandra, NIAS, Bangalore, INDIA
Title: Some Problems of Analytic Number Theory - V

This is a continuation of the paper with the same title but with number IV. Theorem 6 of that paper reads as follows. Let \( \exp(\zeta(s)) = \sum_{n=1}^{\infty} b_n n^{-s} \) and let \( \exp \exp(\zeta(s)) = \sum_{n=1}^{\infty} d_n n^{-s} \) in \( \text{Re}(s) > 1 \). Then we have

\[
\sum_{n \leq x} b_n = \frac{1}{2\pi i} \int_{|s-1|=\frac{10}{11}} \exp(\zeta(s)) x^s \frac{ds}{s} + \Omega(x^{1-\epsilon})
\]

and

\[
\sum_{n \leq x} d_n = \frac{1}{2\pi i} \int_{|s-1|=\frac{1}{10}} \exp \exp(\zeta(s)) x^s \frac{ds}{s} + \Omega(x^{1-\epsilon})
\]

for every fixed \( \epsilon > 0 \). In the present paper we prove that these results are valid even when \( \zeta(s) \) is replaced by \( \alpha(\zeta(s))^{\beta} \) when \( \alpha \) and \( \beta \) are any non-zero complex constants.

Monday, 19 December 2005 (9.30-10.10)

Title: Y. Bugeaud, Universit Louis Pasteur, FRANCE
Title: Linear forms in logarithms of algebraic numbers close to 1 and applications to Diophantine equations

In 1974, Shorey established improved lower bounds for linear forms in logarithms of algebraic numbers when these are real and very close to 1. Roughly speaking, the product of the heights of the algebraic numbers is then replaced by their sum. We will discuss applications of this result to several Diophantine equations, including the equations \((x^k - 1)(y^k - 1) = (z^k - 1)\).
Monday, 19 December 2005 (10.15-10.55)

Speaker: Shanta Laishram, TIFR, Mumbai, INDIA
Title: Powers in arithmetic progression
A well-known theorem of Fermat states that there are no four squares in an arithmetic progression. We shall give several extensions of this result by considering squares in products of integers in arithmetic progression. This is a joint work with T. N. Shorey.

Monday, 19 December 2005 (11.30-12.10)

Speaker: N. Saradha, TIFR, Mumbai, INDIA
Title: On Products with terms in A.P and a term missing
We discuss the problem whether a product with terms in an arithmetic progression and one of the terms missing can be a perfect power.

Monday, 19 December 2005 (12.15-12.55)

Speaker: A. Mukhopadhyaya, Institute of Mathematical Sciences, Chennai, INDIA
Title: Almost squares as product of consecutive integers
Erdos and Selfridge proved that a product of $k(\geq 2)$ consecutive integers is never an $l$-th power for $l \geq 2$. Further they conjectured that a product of $k - 1$ integers out of $k$ consecutive integers is not a square except for two cases. Saradha and Shorey proved this conjecture in affirmative. We try to answer the same question for product of $k - 2$ integers out of $k$ consecutive integers. Some other related questions comes on the way, and then we turn to the case of consecutive terms of arithmetic progression.

Monday, 19 December 2005 (2.30-3.30)

Speaker: Yu. Nesterenko, Moscow State University, RUSSIA
Title: Transcendence, linear forms in logarithms and diophantine equations
I plan to give some exposition of the results of T.N. Shorey connected to areas specified in the title of the talk, with particular emphasis on diophantine equations that were investigated by methods based on estimates of the linear forms in logarithms of algebraic numbers.
Monday, 19 December 2005 (4-5)

Speaker: R. Tijdeman, Leiden University, NETHERLANDS
Title: Shorey’s work on arithmetical progressions
Together with the lecture of Yu. Nesterenko this lecture gives a survey of the many contributions which T.N Shorey has made to number theory, in particular, to the theory on estimates on linear forms in logarithms, and their applications to diophantine equations and related problems.

Tuesday, 20 December 2005 (9.30-10.10)

Speaker: H.M. Stark, University of California, San Diego, USA
Title: To be announced

Monday, 20 December 2005 (10.15-10.55)

Speaker: M. Ram Murty, Queen’s University, CANADA
Title: THE LANG-TROTTER Conjecture
We will apply techniques from analytic, algebraic and transcendental number theory to study the value distribution of Fourier coefficients of normalized Hecke eigenforms as predicted by Lang and Trotter. This is joint work with V. Kumar Murty.

Tuesday, 20 December 2005 (11.30-12.10)

Speaker: Anitha Srinivasan, Indian Institute of Technology, Mumbai, INDIA
Title: On the equation \(x^2 + dy^2 = bk^n\)
We present a classification of all the solutions \((x, y, n)\) of the diophantine equation \(x^2 + dy^2 = bk^n\) where \(\gcd(bk, 2d) = 1\), and \(b\) is represented only by the identity form \(x^2 + dy^2\). Using this classification we solve partially certain equations such as the following. Consider the equation \(x^2 + d = y^n\), where \(d \equiv 3(\text{mod } 4)\) and such that each prime dividing \(d\) is congruent to 3(\(\text{mod } 4\)) and occurs to an odd power. Using our classification we prove that if \((x, y, n)\) is a solution to the above equation, then \(n\) is odd and \(n|3h\) where \(h\) is the class number of the quadratic field \(\mathbb{Q}(\sqrt{-d})\). If \(b\) is a prime power and \(k\) is a prime, then the classification consists of only one class of solutions. We show in this case that there are no solutions when \(b\) and \(k\) satisfy certain conditions. This is a joint work with N. Saradha.
Tuesday, 20 December 2005 (12.15-12.55)

Speaker: K.Srinivas, Institute of Mathematical Sciences, Chennai, INDIA
Title: To be announced.

Tuesday, 20 December 2005 (2.30-3.10)

Speaker: M. A. Bennett, University of British Columbia, CANADA
Title: Integer Points on Congruent Number Curves
We will discuss an explicit characterization of those primes $p$ for which the Diophantine equation

$$y^2 = x(x - 2^ap^b)(x + 2^ap^b)$$

has (nontrivial) solutions in integers. This is joint work with P.G. Walsh.

Tuesday, 20 December 2005 (3.15-3.55)

Speaker: M. Filaseta, University of South Carolina, USA
Title: Different Uses of Diophantine Analysis in the Theory of Irreducibility
Several of T.N. Shorey’s papers provide information about the largest prime factor of a product of consecutive integers. This talk will describe how these papers helped to motivate the speaker’s early investigations into problems associated with the irreducibility of polynomials over the rationals. The talk will then turn to various other Diophantine results and their applications to the theory of irreducible polynomials.

Tuesday, 20 December 2005 (4.30-5.10)

Speaker: M. Nori, University of Chicago, USA
Title: Differential Forms on Algebraic Varieties
A differential form of degree $k$ on a manifold gives rise to a linear functional on the vector space of smooth $k$-chains. Conversely one may ask which linear functionals arise from differential forms. This problem was posed and solved by Whitney. The analogous problems that arise on complex manifolds and algebraic varieties will be discussed.