Vedic Mathematics in Perspective

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"Vedic Mathematics", or rather something that has assumed the name, is spreading widely in the country, especially at schools and in educational circles. It is acquiring proportions of a social phenomenon, and it is high time that its consequences in education and its implications to the intellectual health of the society be examined carefully. It is important to view the phenomenon in perspective, and address any unwelcome effects that could arise, especially from the exaggerated propaganda being carried out.

In the overall context it would be appropriate to begin with the question, what qualifies to be referred as "Vedic"? The Vedas are ancient Indian texts, generally believed to be from the period 1200-500 BCE. They are a repository of knowledge of *those times*. It may also be borne in mind that the Vedic civilisation was formed of pastoral people, organised in village communities. Performance of *yajnas*, in pursuit of material and spiritual goals, is one of the major cultural features that distinguished the Vedic people.

In the strict sense "Vedic" should mean material that is found in the *Vedas*, or something pertaining to them. Material that can be inferred, in a logically satisfactory manner, to be involved in the Vedic civilisation can also be qualify for the adjective "Vedic", so long as its specific relation to the Vedas, or the Vedic civilisation in a broader sense, is set out to clarify the usage of the term.

Even though "Veda" means knowledge, not all knowledge acquired by humans over the course of history, which includes say the discoveries and inventions reported in contemporary journals, can be termed "Vedic". The term "Vedic" inalienably carries with it, in common social perception, a sense of being ancient to an appropriate extent, and it should not be used without confirming to such a norm. This needs to be borne in mind especially in the context of emerging trends in cultural nationalism, some with rather fundamentalist tendencies.

There have been detailed studies of the (genuine) Vedic texts in various respects, and in particular their mathematical contents have been the subject of careful scrutiny by scholars from India as well as the west (see [4], [6], [8]). Decimal representation of numbers, use of zero, geometry involved in the construction of *vedis* and *agnis* involved in the performance of the *yajnas*, are some of the highlights of the period, from a mathematical point of view. The Shulvasutras, which are part of the Kalpasutras from the Vedangas, describe various aspects of construction of the vedis and agnis. Some of these, Baudhayana, Apastambiya, Manava and Katyayana shulvasutras, devote exclusive portions to exposition of geometric constructions of various figures such as the square, rectangle, trapezium, rhombus, circle with the same area as a given square and the other way, etc.. Explicit statements of what is now called "Pythagoras theorem" are found in all the above mentioned shulvasutras. There is also an interesting feature that the converse of the Pythagoras theorem was used extensively for construction of right angles, these being basic to construction of various geometric figures. Baudhayana and Apastambiya describe a value of $\sqrt{2}$ which is remarkably close to the actual value as we now know it.

Now consider the so called "Vedic Mathematics" that is currently in vogue, and touted as an "amazingly compact and powerful system of calculation"; I will refer to it as VM, for brevity as well as for distinguishing it from what should appropriately be called Vedic mathematics, that is briefly discussed above. Let me begin with a brief account of VM. The "system" first made its appearance in print in the book "Vedic Mathematics" [11], first published in 1965, authored by Sri Bharati Krishna Tirtha, who was the Shankaryacharya of the Puri Mutt from 1925 until his passing away in 1960; the book was published posthumously by some of his disciples.

The book is in a narrative style, in English, describing certain devices (not to use the term "trick", usage of which in this context has sometimes met with some needless antagonism) for some arithmetical and algebraic computations, at school level, interspersed with certain strings of a few words in Sanskrit, which the author calls "sutras". The author conveys the impression that the devices he describes are derived from the "sutras", with the latter having the role as cryptic statements towards the former. Any discerning reader can see however that such a relation between the two does not bear out. It is evident from a careful reading of the narrative that Tirthaji had some devices to explain, and as he proceeds with it he simply attaches the "sutra" indicative of some main feature of the device or procedure. For example, when a device involves in the course of its application to a problem, a number "one more than the previous one" in a certain sense, Tirthaji brings in the Sanskrit form of it, "Ekadhikena purvena", as a sutra. The actual idea involved in the device can by no means be deduced from the sutra, and the latter is not a cryptic encapsulation of the idea. Rather it is a tab or a name, which can at best be viewed as being indicative of the main feature of the device. The actual operation of the device involves a lot more than such a main feature on which the tab or name, viz. the sutra, is modelled. The sutras are too brief and devoid of context to intrinsically be a source of information of any kind, as may be seen from the above example.

As to be expected from the above discussion, the so called sutras are not to be found in any Vedic texts, or any ancient text for that matter. All that was involved in introducing them was a rudimentary knowledge of Sanskrit. There is just about 50 words of such text altogether. Also not surprisingly the language of the sutras is not Vedic but in the latter-day Sanskrit style, as was already noted by Dr. Agrawala in the General Editor's Foreword to Tirthaji's book. Actually the tabs could as well have been in any language, as far as their actual function is concerned. Of course, their being in Sanskrit would (and was perhaps aimed to) influence people, but has no contextual significance.

At one level it can be noticed that Tirthaji also does not make a big deal about the sutras themselves. He just throws them in, as and when he comes to the points which he wants to be especially remembered. Depending on the relative importance that he attaches to them he calls them "sutra" or "subsutra". There is no approach, for instance, like "here is a body of sutras that I found in such and such source that I am now going to interpret". He did not even list the "sutras". The list was compiled for the first time by the editors of the book when the book was published posthumously. Though the people involved with the publication seem to piously view it as a mystical package which may have produced more mathematical knowledge if Tirthaji were to live longer, no such sense is found, whether in explicit or implicit form, in Tirthaji's own writing, either in the narrative on the devices or his Preface to the book. Incidentally, even within the book there are things that are not associated with any of the so called sutras, even by way of tagging, so the naive notion of the sutras as a package source of mathematical ideas does not hold good even in the limited context of the book itself.

As to the source of the sutras Tirthaji makes, only once, a vague allusion to a Parisishta (Appendix) to the Atharvaveda, in the Preface. There is no substantiation, description of the original context, or anything else that would give the reference any credibility at all. It has also been noted by Prof. K.S. Shukla, who was a renowned scholar of ancient Indian mathematics that when confronted to show the sutras in a Parisishta to the Atharvaveda, Tirthaji responded saying that it is only in his own Parisishta to the Atharvaveda, and not in other extant Parisishtas! (see [9], [10]). James Glover, an ardent present day proponent of VM and author of a series of books on VM, also confessed in a letter to me in 1994: "Considering the style of Sanskrit used in the sutras, together with the report that they were found only in his Parishishta, it seems more likely that he discovered the sutras himself.". Taken together with the fact that the whole of Tirthaji's preface is full of illogical pious presumptions such as Vedas contain *all* knowledge, it is clear that the claim of a Vedic source is not valid in the conventional sense and it does not even meet common sense criteria.

The computational devices that Tirthaji exposes do not have anything to do with the Vedas either. Several concepts crucial to the devices exposed in VM are well outside the domain of the Vedic civilization. The decimal point (fractional) representations, involved in the first trick, came to be introduced only in the sixteenth century. There are no decimal fractions in the works of Bhaskaracharya from the twelfth century, leave alone in Vedic times. Many other operations involved also have no relation to ancient Indian mathematics, all the way from the Shulvasutras from the Vedic times, to the mathematics from Kerala in the sixteenth century; see [2] and [3] for detailed comments on the mathematical contents of Tirthaji's book; in passing I may point out here that Tirthaji's book shows acute lack of awareness of the genuine ancient mathematics, given especially the context that the author was trying to be superlative about ancient Indian achievements.

One of the things that seems to have persuaded people to take VM as ancient is the feeling that there is no other possible source. It is generally not realised that the devices of VM are well within the scope of contemporary mathematics. Several books have been written over the last century, both in India and abroad, describing tricks for performing various special arithmetical operations. The Trachtenberg speed arithmetic, Lester Meyers' arithmetic are some well-known examples; see [1], [7]. (Many readers who are old enough may also have seen in the 1950's books by Indian authors, as I did during my childhood, dealing with arithmetic in unconventional ways, but unfortunately it is hard to trace them; efforts to bring them to light would be worthwhile.) Tirthaji's book is on a similar footing, in terms of mathematical significance. It only distinguishes itself by the baseless claim of an ancient source. While the devices of VM are interesting, there is no big surprise or mystery about them, from the point of view of the twentieth century general knowledge in mathematics. Putting together such a compilation was entirely within the realm of overall individual capabilities in the twentieth century. It may also be noted that Tirthaji had advanced mathematical training in the twentieth century format and had obtained a Masters degree in mathematics. It may also be noted that Manjula Trivedi, a disciple of Tirthaji involved with the publication of Tirthaji's book, mentions in her introductory write-up included in the book, about eight years of "Tapas" at Shringeri by Tirthaji in connection with the genesis of the book.

The kind of computations dealt with by Tirthaji could not have been of frequent occurrence until the twentieth century. Tirthaji's exposition of the devices (perhaps only some to begin with) came in the first half of the twentieth century. They may in fact have come into general use just around then, and Tirthaji may have been the first expositor. While Tirthaji indeed was more comprehensive in the matter, it is certain that many of the devices were known to other people of his times. I was once asked of Tirthaji "if it was his own creation would he not have taken credit for it rather than calling it Vedic". The answer lies in this broad context and the fact that the work was largely of compilation.

Having discussed the issue ancient source (or rather the absence of it), and put material in perspective let me now come to the aspect of usefulness of VM.

It may be emphasized that the only use of the devices of VM is to help rapid calculations in certain special situations. In particular the main aspects of clarification of concepts, logical thinking etc. which are the crucial needs in respect of the mathematical training of children are not served by VM. The conventional system, whatever its shortcomings, is the result of a cumulative understanding and effort in achieving these objectives. Preoccupation with VM can therefore be positively harmful to the overall growth of a child's mathematical talents, as it would create a warped perspective in which a bit of speed and flash, in what are actually unrepresentative situations, would get encouraged at the cost of basic understanding of mathematical ideas.

Teachers of VM have sometimes narrated their experience that even after being taught the devices of VM and practising them in the course of the training, outside the programme the student do not tend to use them. Though the teachers blame it on excessive hold of the conventional system, the real reason, at a psychological level, possibly rests in the fact the conventional way of doing the computations is dependable and straightforward, and does not involve recognising the patterns which would make one or other device from VM applicable, which is taxing to the child's mind. It may also be noted that contrary to the perception that VM involves doing calculations creatively, in practice it involves it own rote which further has the disadvantage that the procedures to be followed are several steps away from the logical framework involved, compared to the conventional ways.

Till the nineteen-eighties VM had a limited response, and in a way the limitations of it seem to have been recognised. In the eighties some English school teachers got involved with VM. It then became a rallying point for its supporters that VM is appreciated in the west. The background of the groups promoting VM in the outside world was rarely taken into account. For instance the St. James Independent School, London which is often quoted in this context, is a school run by the 'School of Economic Science' which is, according to a letter I got from Mr. James Glover, the Head of Mathematics at the School, "engaged in the practical study of Advaita philosophy". The claims of appreciation abroad can thus be very misleading. Over the recent years these groups have played an increasing role in promoting VM, and their impact is felt in India as well. Some of the more commonly used books for VM are by English authors (who curiously are favoured over their Indian counterparts, even by the Indian publishers), though their intrinsic merits from a pedagogical point of view have hardly been critically examined in the educational circles. I may mention that in fact some serious lacunae have been pointed out in these books by Dr. K. Subramanian of the Homi Bhabha Centre for Science Education, Mumbai, with regard to conceptual issues, in comparison with the conventional textbooks in India.

It should also be borne in mind that being able to calculate rapidly has lost whatever importance it had, with the advent of calculators. It is now like learning to run fast or jump very high. While these would indeed bring benefits on occasion, they are worthwhile to pursue only when one has either a natural flare for it, or one's particular context warrants it in some way. The utility of any training needs to be measured against the time and effort that it would take on the part of the trainee and the trainer, and from this perspective burdening typical innocent children with VM is highly inadvisable, and harmful in the longer run.

When attaining proficiency in rapid arithmetic is a goal, VM is really but one stream (and perhaps more boring one on account of the preoccupation of its purveyors with building up emotions rather than focusing on the ideas themselves). There are also other systems such as the Trachtenberg speed arithmetic from the 1950's and its later day variants; see [5] for instance. One ought to take a comprehensive view and see what the best way towards attaining the objective would be, and not be drawn into carrying the burden of excess baggage of the so called "ancient wisdom", which is only a myth.

References

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